Coherence, space non-locality and lacunarity in a cascade model of turbulence

Piero Olla[1] and Paolo Paradisi[2]

[1]ISIAtA-CNR, 73100 Lecce Italy
[2]FISBAT-CNR, 40129 Bologna Italy

Abstract

We use multiscale-multispace correlations and Fourier transform techniques, to study some intermittent random field properties, which escape analysis by structure function scaling. These properties are parametrized in terms of a set of scale ratios, giving the typical interaction distances in space and scale of the random field fluctuations, and the characteristic lengths over which these fluctuations act coherently to generate intermittency. The relevance of these techniques in turbulence theory is discussed.

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Turbulence is usually depicted as a situation of irregular fluid motion, with energy being transferred from large to small scales, by interaction between eddies of increasingly smaller size. This mechanism of energy transfer, usually referred to with the name of cascade, takes different forms depending on the choice of turbulence description. In closure [1], a Fourier representation of the Navier-Stokes equation is typically used. In this case, the transfer of energy between eddies is expressed in terms of an energy flux in wavevector space. This flux is a nonlinear kernel of the spectral energy density, and therefore a fully global quantity, carrying no information on spatial structure and transfer fluctuations. In intermittency modelling, instead, the description is necessarily local in space, at least implicitly. In the random beta model [2], for example, this is obtained using a superposition of nested eddies of increasingly smaller size, with a fluctuating eddy to eddy energy transfer.

In both circumstances, the dynamics forces correlations between the fluctuations at different scale and space locations. Only recently, however, there has been some interest in studying correlations in turbulence [3], [4], beyond the standard approach of focusing on structure function scaling.

A tool that is receiving great attention, especially in the study of intermittency, is synthetic turbulence [5], [6], [7]. Synthetic turbulence has no pretense of modelling turbulence dynamics, its goal being more the one of reproducing the statistics observed in experimental turbulence data. Even so, the multiplicative noise algorithms used to produce the turbulent signal contain implicit assumptions on the nature of the turbulent flow that go beyond the task of reproducing anomalous structure function scaling.

Typically, these synthetic signals are expressed as a superposition of wavelets with random coefficients:

$$\Psi(x) = \sum_{n}^{N} \sum_{\mathbf{h}_n} A_{\mathbf{h}_n} w_S(k_{\mathbf{h}_n}, y_{\mathbf{h}_n}, x), \tag{1}$$

where $\Psi(x)$ is the synthetic turbulent signal, with $w_S(k, y, x)$, wavelets of scale k and position y [8], and the vector index $\mathbf{h}_n = (h_0, h_1, ...h_n)$ identifying the position of the wavelet in the cascade through the sequence of its ancestors: the integer h_n labels the h_n-th daughter wavelet generated at the n-th step in the cascade, by wavelet $\mathbf{h}_{n-1}$. The standard practice [6] is to arrange the wavelets on a tree structure in $k - y$ space, with $k_{\mathbf{h}_n} \equiv \bar{k}_n = \bar{k}_0 2^n$, $h_i = \pm 1$ $y_{\mathbf{h}_n} = y_{\mathbf{h}_{n-1}} + \frac{h_n}{2k_n}$. The amplitudes $A_{\mathbf{h}_n}$ are then generated through a random multiplicative process.

This picture closely parallels the one of a random beta model [2]; the resulting correlation pattern has clearly a multifractal (multiaffine) nature, depending on the singularity (lack of singularity) of $\Psi(x)$, as $N \to \infty$ in Eqn. (1). The difference, in the case of the beta model, is that the multiplicative random process, arises explicitly out of the assumption of an energy transfer, local in both space and scale.

The purpose of this letter is to present some techniques, based on the use of wavelet and Fourier analysis, to extract informations on the algorithm of signal generation, directly from the signal statistics. We focus in particular on two important aspects: the determination of which "building block" wavelet w_S is being used in the synthetic signal, and which is the typical interaction distance between wavelets, as expressed by the separation in $k - y$ space: $(k_{\mathbf{h}_n} - k_{\mathbf{h}_{n-1}}, y_{\mathbf{h}_n} - y_{\mathbf{h}_{n-1}})$. In real turbulence, these aspects have a direct interpretation in terms of eddy structure and energy transfer properties.

The choice of the wavelet, identifies a cell of $k - y$ space, which acts coherently to generate intermittency; this is in practice determined by the "number of wiggles" of the wavelet, i.e. the product $a_S = k\lambda_S$ of its characteristic wavevector and spatial extension. We can thus focus on the case in which w_S is a Gaussian wavepacket, without fear of loosing to much in generality:

$$w_S(k, y, x) = \exp(ik(x - y) - (x - y)^2/\lambda_S^2). \tag{2}$$

The interaction distance provides informations on the degree of non-locality of the dynamics, by telling us whether only nearest neighbor cells in $k - y$ space can interact directly, or longer range

interactions are allowed. The algorithms of turbulence synthesis studied in [6], [3] and [4] are strictly local. To allow for non-locality, the simplest approach is to treat the wavelet coordinates in $k-y$ space, on the same ground as the amplitudes $A_{\mathbf{h}_n}$, as stochastic variables. In this way, the wavelets take more the aspect of eddies, with complete freedom of location (and overlap) in $k-y$ space [9]. The dynamics is described through the probability for the transition $(\ln A_{\mathbf{h}_{n-1}}, \ln k_{\mathbf{h}_{n-1}}, y_{\mathbf{h}_{n-1}}) \to (\ln A_{\mathbf{h}_n}, \ln k_{\mathbf{h}_n}, y_{\mathbf{h}_n})$, which is taken in the form: $p_A(A'/A|k'/k)p_k(k'/k)p_x(k|y-y'|)$, with $\int \mathrm{d}\ln y\, y^p \times p_A(y|x) = c_p x^{-\zeta_p}$ to insure power law scaling in k for the moments of A. The phases are supposed random so that intermittency in Ψ arises simply as a consequence of the intermittency of the amplitudes A. We take for simplicity, p_x and p_k to be Gaussians respectively in space separation and $\ln k_{\mathbf{h}_n}/k_{\mathbf{h}_{n-1}}$. The transition probabilities over n cascade steps will be Gaussians as well:

$$P_x(n,k|y-y'|) \simeq \frac{k}{\pi^{\frac{1}{2}}\hat{b}a_S}\exp\Big(-\frac{k^2|y-y'|^2}{\hat{b}^2a_S^2}\Big)$$

$$P_k(n;k'/k) = \frac{1}{(\pi n)^{\frac{1}{2}}\Delta z}\exp\Big(-\frac{(\ln(k'/k)-n\bar{z})^2}{n\Delta z^2}\Big). \tag{3}$$

with $\hat{b}$ giving the amount of space non-locality and $\Delta z/\bar{z}$ the degree of discreteness in scale, in the cascade algorithm. Thus, the wavelet wavevectors in the cascade will be centered at $\bar{k}_n = \bar{k}_0\exp(n\bar{z})$ with increasing width $\Delta\bar{k}_n \sim n^{\frac{1}{2}}\Delta z\bar{k}_0$. The probability p_A, which is the multiplier distribution for the coeffincients $A_{\mathbf{h}_n}$, can be obtained starting from the distribution of the scaling exponents ζ_q, using standard techniques [10]. We do not need its explicit form here, however.

To measure effects such as the degrees of interaction non-locality and discreteness, in an intermittent signal of unknown origin, multiscale-multispace wavelet correlations would be clearly the instrument of choice [3], [4]. This is conditioned, however, to having some knowledge of the structure of the building block wavelets w_S, especially, as regards the value of the parameter a_S.

The simplest wavelet correlations are $\langle|\Psi_{ky}|^2\rangle$ and $\langle|\Psi_{ky}|^2|\Psi_{k'y'}|^2\rangle$, where Ψ_{ky} is the component of Ψ on a wavelet $(-k)^{-s}\partial_x^s w_A(k,y,x)$. Here, w_A is taken again to be a Gaussian wavepacket, with $a_A = k\lambda_A$ not necessarily equal to a_S, and s is chosen large enough to kill large scale contributions to Ψ:

$$\Psi_{ky} = \lambda_A^{-1}k^{-s}\int \mathrm{d}x\, w_A^*(k,y,x)\partial_x^s\Psi(x). \tag{4}$$

In a cascade model like the one considered here, the wavelet components Ψ_{ky} entering $\langle|\Psi_{ky}|^2 \times|\Psi_{k'y'}|^2\rangle$ may come from a single building block wavelet w_S ("one-eddy" contribution), if $|k-k'|$ and $|y-y'|$ are both small enough; otherwise they will come predominantly from two distinct w_S placed at (k,y) and (k',y') ("two-eddy" contribution). If $|y-y'|<L$, with L the largest scales in the signal, there will be a correlation between the two building block wavelets w_S, due to their coming from some common ancestor at scale $\hat{k}\sim\hat{b}a_S|y-y|^{-1}$. This situation is expressed through the formula:

$$\langle|A_{\mathbf{h}_n}|^2|A_{\mathbf{h}'_m}|^2\rangle = c_4(k_{\mathbf{h}'_m}L)^{-\zeta_4}(k_{\mathbf{h}_n}/k_{\mathbf{h}'_m})^{-\zeta_2}(k_{\mathbf{h}'_m}/k_{\mathbf{h}_p})^{\zeta_4-2\zeta_2} \tag{5}$$

with $k_{\mathbf{h}_n} > k_{\mathbf{h}'_m}$ and p the cascade step at which the genealogical tree of $\mathbf{h}_n$ and $\mathbf{h}'_m$ branches: $h_i = h'_i$ for $0\le i\le p$ and $h_i\neq h'_i$ for $i>p$. Thus, the lower the branching takes place in the tree, the closer the correlation gets to its disconnected limit $\langle|A_{\mathbf{h}_n}|^2|A_{\mathbf{h}'_m}|^2\rangle \sim (k_{\mathbf{h}'_m}k_{\mathbf{h}_n})^{-\zeta_2}$.

Indicate with $C(k,y;k',y')$ the square modulus of the component of w_S with respect to w_A:

$$C(k_A,y_A;k_S,y_S) = \Big|\lambda_A^{-1}k_A^{-s}\int \mathrm{d}x\, w_A^*(k_A,y_A,x)\partial_x^s w_S^*(k_S,y_S,x)\Big|^2$$

$$\simeq \frac{a_S^2}{a_A^2+a_S^2}\exp\Big(-\frac{2k_A^2}{a_A^2+a_S^2}\Big(\Delta y^2+\frac{a_A^2a_S^2\Delta k^2}{4k_A^2}\Big)\Big), \tag{6}$$

where $\Delta k = k_A - k_S$ and $\Delta y = y_A - y_S$. We can now write down the expression for the second order correlation. From Eqn. (3), for $\ln kL > \bar{z}^3/\Delta z^2$, discreteness effects can be neglected, and we have, indicating with $\langle P_k(n, \bar{k}/\bar{k}_0)\rangle$, average over the size of the large scale building block wavelets:

$$\langle|\Psi_{ky}|^2\rangle = c_2 \int \mathrm{d}\bar{y}\mathrm{d}\ln\bar{k} \sum_n \bar{k}_n \langle P_k(n, \bar{k}/\bar{k}_0)\rangle (\bar{k}L)^{-\zeta_2} C(k, y; \bar{k}, \bar{y}) \simeq \frac{c_2 \pi a_S}{\bar{z} a_A} (kL)^{-\zeta_2'}. \tag{7}$$

where the difference $\epsilon = \zeta_2' - \zeta_2 = \mathcal{O}(\bar{z}^{-3}\Delta z^2)$ is due to the fluctuation in the scale ratio between a daughter wavelet $\mathbf{h}_n$ and her parent $\mathbf{h}_{n-1}$, and we have used the result, valid for $\ln k/\bar{k}_0 > \bar{z}^3/\Delta z^2$: $\sum_{n=1}^{\infty}(\bar{k}_n/k) P_k(n, k/\bar{k}_0) \simeq \bar{z}^{-1}(k/\bar{k}_0)^{\epsilon}$. We have a similar expression for the one-eddy contribution to the 4-th order correlation:

$$\langle|\Psi_{ky}|^2|\Psi_{k'y'}|^2\rangle_1 \simeq \frac{c_4 \pi a_S}{\bar{z} a_A} (kL)^{-\zeta_4'} (k'/k)^{-\zeta_2'} C(k, y; k', y') \tag{8}$$

From Eqns. (8) and (6), if the space-scale separation is large enough, $\langle|\Psi_{ky}|^2|\Psi_{k'y'}|^2\rangle$ will be dominated by the two-eddy contribution. In this case, however, it is not sufficient to impose that kL be large, to avoid discreteness effects. Limiting the calculation to the connected part of the correlation, we find:

$$\langle|\Psi_{ky}|^2|\Psi_{k'y'}|^2\rangle_c = 2c_4 \int_{\bar{k}'>\bar{k}} \mathrm{d}\ln\bar{k}\mathrm{d}\ln\bar{k}'\mathrm{d}\bar{y}\mathrm{d}\bar{y}' \int_{\ln L^{-1}}^{\ln k} \mathrm{d}\ln\hat{k}\, (\bar{k}L)^{-\zeta_4} (\bar{k}'/\bar{k})^{-\zeta_2} (\bar{k}/\hat{k})^{\zeta_4 - 2\zeta_2}$$
$$\times \sum_n \sum_{m=0}^{n} \sum_{p=1}^{m} \frac{\bar{k}_n \bar{k}_m}{\bar{k}_p} \langle P_k(p, \hat{k}/\bar{k}_0)\rangle P_k(n-p, \bar{k}/\hat{k}) P_k(m-p, \bar{k}'/\hat{k})$$
$$\times P_x(\hat{k}|\bar{y} - \bar{y}'|) C(k, y; \bar{k}, \bar{y}) C(k', y'; \bar{k}', \bar{y}'). \tag{9}$$

where $(\bar{k}_m/\bar{k}_p) P_x(\hat{k}|\bar{y} - \bar{y}'|)$ is the space density at $\bar{y}'$, of wavelets $\mathbf{h}_m'$ generated from the branching at $\mathbf{h}_p$, given the presence of a wavelet $\mathbf{h}_n$ at $\bar{y}$, and $\bar{k}_n$ is the space density of wavelets $\mathbf{h}_n$. From here, informations on the space-scale structure of the intermittent signal can in principle be obtained. The slow decay of correlations between wavelets, shown in Eqn. (5), however, suggests that the effect of coherency, important for $k|y - y'| < a_S$, and the one of interactions, important for $k|y - y'| < \hat{b} a_S$, will be superimposed in a way that is difficult to disentangle. This warns against trying to measure a_S looking for crossovers in the space scaling of correlations. We have in fact two mechanisms of correlation distruction: one is the decay of the one-eddy contribution, due to the two analyzing wavelets ceasing to overlap with a single w_S. The second is the decay of the two-eddy contribution, i.e. the decrease with distance of the correlation between different w_S. Only the first effect, of course, gives informations about the shape of w_S.

Surprisingly, this strongly local information is more easily obtained using a fully global instrument like a Fourier transform, at least if the random phase hypothesis is satisfied. The reason is the maximal definition in scale of a Fourier component. In this case, a correlation like $\langle\Psi_{-k}^2 \Psi_{k-\Delta}\Psi_{k+\Delta}\rangle$, with Ψ_k standard Fourier components, will be non-zero only when $\Delta < \lambda_S^{-1} = k/a_S$; in fact, a 4-th order correlation can receive contributions at most by two w_S (plus their complex conjugate), which cannot cover a section of k including at the same time $-k$, $k - \Delta$ and $k + \Delta$, when Δ is large. In the example of our cascade model, we can calculate this fourth order correlation rather easily. In the case of negligible discreteness effects, we find:

$$\langle\Psi_{-k}^2 \Psi_{k-\Delta}\Psi_{k+\Delta}\rangle \simeq \frac{\pi^{\frac{5}{2}} a_S^4 \delta(0) k^{-3-\zeta_4'}}{\bar{z}\alpha} \Big[1 + \frac{4\pi k}{\mathbf{e}\hat{b} a_S \bar{z}^2 \alpha \Delta} \exp\Big(-\frac{a_S^2 \Delta^2}{4k^2}\Big)\Big] \exp\Big(-\frac{a_S^2 \Delta^2}{4k^2}\Big) \tag{10}$$

where $\alpha = (2 + \zeta_2' + a_S^2/2)^{\frac{1}{2}}$ and the two terms in square brackets are respectively the one- and two-eddy contribution to the correlation.

Once a_S is known, it is possible to set $a_A = a_S$ in the analyzing wavelet, in order to maximize the overlap between the analyzing wavelets w_A and the coherent regions identified by the building block wavelets w_S. At this point it is possible to look at the dependence on space and scale separation, of the expression for the correlation $\langle|\Psi_{ky}|^2|\Psi_{k'y'}|^2\rangle_c$ provided by Eqn. (9). We consider first the case in which discreteness effects can be neglected, corresponding to the regime $\ln k'/k > \bar{z}^3/\Delta z^2$. Setting $a_A = a_S$ we obtain:

$$\langle|\Psi_{ky}|^2|\Psi_{k'y'}|^2\rangle_c \simeq \frac{2c_4\pi^{\frac{3}{2}}}{\bar{z}^3\hat{b}a_S}(kL)^{-\zeta_4'}(k'/k)^{-\zeta_2'}\int_0^{\ln kL}\mathrm{d}x\Big(1+A\mathbf{e}^{-2x}\Big)^{-\frac{1}{2}}$$

$$\times\exp\Big[(\zeta_4'-2\zeta_2')x-\Big(\mathbf{e}^{2x}+F\Big)^{-1}\frac{k^2|y-y'|^2}{\hat{b}^2a_S^2}\Big] \tag{11}$$

where $F = \hat{b}^{-2}(1+(k/k')^2)$. From inspection of this equation, we see that the integral receives contribution, for $\hat{b}a_S$ large enough, from $\max(0, \ln\frac{k|y-y'|}{\hat{b}a_S}), \ln F) < x < \ln kL$, where the integrand is essentially $\exp((\zeta_4'-2\zeta_2')x)$. We find then, for $2\zeta_2'-\zeta_4'$ small:

$$\langle|\Psi_{ky}|^2|\Psi_{k'y'}|^2\rangle_c \simeq \frac{2c_4\pi^{\frac{3}{2}}}{\bar{z}^3\hat{b}a_S}(kL)^{-\zeta_4'}(k'/k)^{-\zeta_2'}\max\Big(\ln GkL,(2\zeta_2'-\zeta_4')^{-1}\Big)G^{2\zeta_2'-\zeta_4'} \tag{12}$$

where $G = \max(1, \frac{k|y-y'|}{\hat{b}a_S}, F^{\frac{1}{2}})$. We thus have a close range and a large separation range, with transition at $|y-y'| \sim (a_S/k)\max(\hat{b},(1+(k/k')^2)^{\frac{1}{2}})$. In the close range, the correlation scales only in k: $\langle|\Psi_{ky}|^2|\Psi_{k'y'}|^2\rangle_c \sim (kL)^{-\zeta_4'}(k'/k)^{-\zeta_2'}\ln kL$, while, in the large separation regime, the correlation contains a factor scaling like a power in $|y-y'|$: $\langle|\Psi_{ky}|^2|\Psi_{k'y'}|^2\rangle_c \propto |y-y'|^{\zeta_4'-2\zeta_2'}$. This slow decay in $|y-y'|$, observed also in [3], and the one of structure functions, with respect to a_A [9], have the same origin in the dependence on $k_{\mathbf{h}_p}$ of $\langle|A_{\mathbf{h}_n}|^2|A_{\mathbf{h}_m'}|^2\rangle$ [see Eqn. (5)]: larger space separations between building block wavelets imply a lower branching in their genealogical tree.

Thus, if $\hat{b}$ is large, for any scale k there is a second characteristic length, beyond λ_S, which gives the degree of space non-locality in the interaction, and which can be measured by locating the crossover to power law scaling with respect to $|y-y'|$, in $\langle|\Psi_{ky}|^2|\Psi_{k'y'}|^2\rangle$.

If $\bar{z}^3/\Delta z^2$ is large enough, there is a range of scale separations: $\ln k'/k < \bar{z}^3/\Delta z^2$ where discreteness effects are relevant. In this regime, only one term contributes in the sums over m and n in Eqn. (9): the one corresponding to the closest $\bar{k}_n$ and $\bar{k}_m$ respectively to $\bar{k}$ and $\bar{k}'$. (The sum over p remains continuous, provided kL is large). Setting again $a_A = a_S$, the result, after some lengthy algebra, is equivalent to the one of Eqn. (11):

$$\langle|\Psi_{ky}|^2|\Psi_{k'y'}|^2\rangle_c \simeq \frac{2c_4\pi^{\frac{3}{2}}\bar{z}^{\frac{1}{2}}}{\Delta z\hat{b}a_S}\int_0^{\ln kL}\mathrm{d}x\,H^{-\frac{1}{2}}\Big(1+F\mathbf{e}^{-2x}\Big)^{-\frac{1}{2}}$$

$$\times\exp\Big[(\zeta_4'-2\zeta_2')x-\Big(\mathbf{e}^{2x}+F\Big)^{-1}\frac{k^2|y-y'|^2}{\hat{b}^2a_S^2}-\frac{\bar{z}^3\Delta_{kk'}^2}{2H\Delta z^2}\Big] \tag{13}$$

where $H = 2x+\ln k'/k+\frac{4\bar{z}^2}{\Delta z^2a_S^2}$ and $\Delta_{kk'} = \bar{z}^{-1}\ln k'/k - \mathrm{int}(\bar{z}^{-1}\ln k'/k)$ is the decimal part of $\bar{z}^{-1}\ln k'/k$. The main difference from Eqn. (11) lies in the quadratic term in $\Delta_{kk'}$ in the exponential. As in the case of Eqn. (11), the integral receives contribution for $\ln G < x < \ln kL$. The integrand has a saddle at: $\bar{x} \sim a_S(\bar{z}/2)^{\frac{3}{2}}(2\zeta_2-\zeta_4)^{-\frac{1}{2}}|\Delta_{kk'}|-2\bar{z}^2$ which starts playing a role, however, only at very small values of $\Delta_{kk'}$. Thus the integral can be estimated almost always with steepest descent at $x \sim \ln G$. The result is:

$$\langle|\Psi_{ky}|^2|\Psi_{k'y'}|^2\rangle_c \simeq \frac{2c_4\pi^{\frac{3}{2}}\bar{z}^{\frac{1}{2}}}{\Delta z\hat{b}a_S}(kL)^{-\zeta_4'}(k'/k)^{-\zeta_2'}G^{2\zeta_2'-\zeta_4'}$$

$$\times \max\Big(\ln GkL, (2\zeta_2' - \zeta_4')^{-1}\Big) \exp\Big(-\frac{\bar{z}^3\Delta_{kk'}^2}{2\Delta z^2(\ln G + (\frac{2\bar{z}}{a_S\Delta z})^2)}\Big) \tag{14}$$

which differs from Eqn. (12), again because of the term in $\Delta_{kk'}$. This produces oscillations in the correlation dependence on $\ln k'/k$, (lacunarity), which have period $\bar{z}$, and die when $\ln k'/k$ or $\ln \frac{k|y-y'|}{\hat{b}a_S}$ become larger than $\bar{z}^3\Delta z^{-2}$. This at least in the physically interesting regime where the uncertainty in the wavevector over a cascade step is of the same order or larger than the spectral width of w_S.

Dynamically, what happens is that the correlation between w_S at a scale separation $\ln k'/k$ which is not an integer number of $\bar{z}$, is due to a common ancestor at a scale $\hat{k}$, which must be distant a sufficient number n of cascade steps from k. This is necessary for the width $n^{\frac{1}{2}}\Delta z$ to cover the difference $\Delta_{kk'}$ between $\bar{z}^{-1}\ln k'/k$ and the next integer. Clearly, the greater the number of cascade steps, the smaller the contribution to the correlation, and this explains the decrease in $\Delta_{kk'}$, of $\langle|\Psi_{ky}|^2|\Psi_{k'y'}|^2\rangle_c$, shown in Eqns. (13-14).

We thus have three additional characteristic parameters describing the structure of a cascade generated random field: the oscillation period in $\ln k'/k$ of the correlation, and the separation in space and scale over which these oscillations die off. As regards turbulence modelling, it is worth mentioning, that different values of $\bar{z}$ correspond to different degrees of non-locality in the Fourier structure of the Navier Stokes equation energy transfer terms. Analysis carried on by means of direct numerical simulations and closure has shown indeed that such non-local effects may be an important component of turbulence dynamics [11].

Summarizing, for any fluctuation scale k we have up to five additional scales at our disposal, to characterize an intermittent random field. This beyond the standard spectrum of scaling exponents, provided by structure function analysis. It is worth considering that these quantities have been introduced through an operative definition, which allows their measurement independently of the signal being synthetic, and originating from a random cascade. In particular, even in the case of a generic random field, in which one would expect the coexistence of fluctuations of different shapes, a concept like the coherence lenght λ_S is going to maintain its physical meaning, at least on an average sense. The same holds for all the quantities obtained from the multiscale-multispace correlation $\langle|\Psi_{ky}|^2|\Psi_{k'y'}|^2\rangle$: the degree of space non-locality, identified by the crossover to scaling in the space separation, the lacunarity period, measured through the oscillation in $\ln k'/k$, and the separation in scale and space for the decay of these oscillations. This suggests, that methods for the characterization of intermittent signals, like the ones presented here, could be of some interest also for people with access to experimental high Reynolds numbers turbulence data.

From the point of view of turbulence modelling, it is interesting to examine in more detail what happens in some limiting cases.

A first limit is obtained when $\hat{b} = L/\lambda_S$, and corresponds to the maximum degree of space non-locality in the interaction. In this regime, $F \sim (\lambda_S/L)^2$ and $\langle|\Psi_{ky}|^2|\Psi_{k'y'}|^2\rangle_c \sim (kk')^{-\zeta_2'}$, which means that the wavelets w_S are essentially uncorrelated, although intermittently distributed; in fact, the one-eddy contribution leads still to the anomalous structure function scaling: $\langle|\Psi_{ky}|^4\rangle \sim (kL)^{-\zeta_4'}$. This situation corresponds to the random eddy model, studied in [12].

A different, more interesting kind of space non-locality is obtained when $\lambda_S \sim L$, which implies that $a_S = k\lambda_S \to \infty$ as $k \to \infty$. In this case, the building blocks of the random field are not wavelets anymore, but Fourier components. We would have then a signal generated by a superposition of random phase Fourier modes, analogous to the turbulence picture one gets out of statistical closures. The only difference would be the increasing intermittency of the Fourier amplitudes as $k \to \infty$. However, as we could expect, this does not produce any intermittency in the signal; in fact, it can be shown, by generalizing the result of Eqns. (8), (12) and (14), to the case of a_A fixed and a_S large, that: $\langle|\Psi_{ky}|^2\rangle^{-2}\langle|\Psi_{ky}|^4\rangle_c \propto a_S^{-1}$. Hence, the non-Gaussian contribution to the correlation decays like k^{-1} as $k \to \infty$.

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